would increase the measurement time somewhat. Another approach to increased accuracy would be to model the diodes and mathematically correct for their nonlinearity with the computer. This latter approach is presently under investigation.

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# Automated Calibration of Directional-Coupler–Bolometer-Mount Assemblies

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*Abstract*—**Although the application of automated methods to power calibration problems in the UHF and microwave region has been described by a number of authors, the primary orientation has been towards the calibration of bolometer mounts and similar items. Little has been published on the problem of calibrating directional-coupler–bolometer-mount assemblies, which also play a major role in the calibration and measurement of UHF and microwave power.**

**This paper develops a theoretical basis for several different approaches to this measurement problem.**

## I. BACKGROUND

MICROWAVE power calibrations tend to center around devices of two basic types. The first is the terminating power meter, a common example of which is the bolometer or thermistor. The second basic device is the feedthrough power meter which often takes the form of a directional coupler with a power meter (frequently of the bolometric type) attached to its sidearm. Although a number of authors [1]–[3] have described the application of automated techniques to the calibration of bolometer mounts, little has been done in the area of automating the calibration of directional-coupler–bolometer-mount assemblies. It is to this problem that this paper addresses itself.

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## II. INTRODUCTION

As already noted, it is possible to regard the directional-coupler–power-meter assembly as a feedthrough power meter. It is perhaps more instructive, however, to visualize the assembly in the context of Fig. 1, where a feedback loop is employed to maintain the sidearm power at a constant level. Under this mode of operation, the Thevenin equivalent generator, which obtains at the coupler output port, has a source impedance which is usually close to a $Z_0$ (reflectionless) match and, in any case, depends *only* on the directivity and other properties of the coupler [4]. The voltage associated with the equivalent generator is

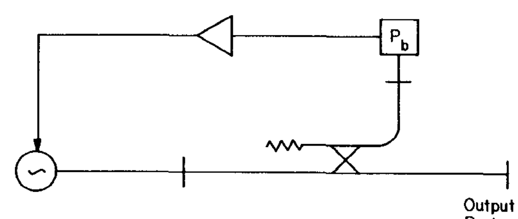


Fig. 1. Basic circuit for discussion of calibration problem.

proportional to the square root of the power level at which the sidearm is maintained by the feedback loop. Here, the proportionality factor depends *only* upon the properties of the coupler *and* sidearm power meter. In the absence of feedback, it is convenient to represent the operation by an equivalent circuit where the generator is of *fixed* source impedance and *variable* voltage, but which may be monitored by observing the variations in sidearm power level. A measurement of some or all of the properties of the coupler assembly, which determine the parameters of the equivalent generator, is usually the object of a calibration of this type of power meter.

Instead of the Thevenin equivalent, it is more common at microwave frequencies to characterize a generator either in terms of its available power, or in terms of the net power which it would deliver to a nonreflecting load. Perhaps the most common measurand in this context has been the "calibration factor" $K_c$, which is defined as the ratio[1] of the power delivered to a nonreflecting load[2] to the power indicated by the sidearm power meter $P_b$. A more recently proposed measurand, $K_A$, is defined as the ratio of the available power at the output port to the sidearm power. The conversion between these methods of characterizing the generator involves only the impedance of the Thevenin equivalent generator. This impedance, or the corresponding reflection coefficient, is also of considerable practical interest.

A complete description of the directional coupler–power meter involves nine parameters (e.g., one real and four complex numbers). The preceding discussion has identified three parameters, $K_c$ (or $K_A$), and the complex reflection coefficient, $\Gamma_g$, of the equivalent generator. The six remaining parameters can be expressed as the three complex numbers which relate the wave amplitudes at the input port to those at the output port. In many cases, these parameters are of limited interest. If required, they may be measured by existing techniques for a low-loss two-port [5].

## III. REVIEW OF PRIOR ART

A very simple calibration procedure is to connect a "standard" nonreflecting power meter to the output port of the system in Fig. 1 and compare its reading with that at the sidearm. This leads to a direct determination of $K_c$. As a practical matter, existing power meters rarely satisfy the nonreflecting stipulation. Fortunately, however, the properties of the directional coupler are often such as to provide a close approximation to a $Z_0$ matched generator. By calibrating the power meter in terms of incident rather than net power (resulting in "cal" factor rather than effective efficiency) one is able to make an approximate determination of $K_c$ by this technique.

Perhaps next in order of complexity is the system

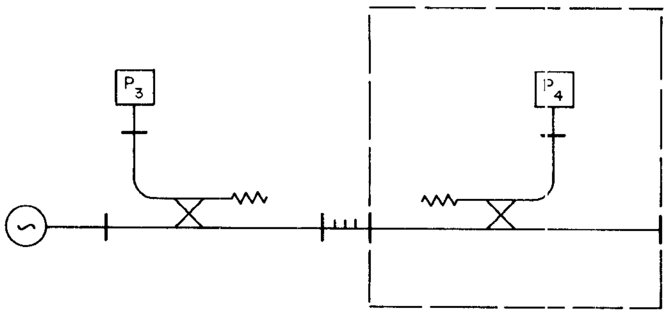


Fig. 2. Illustration of an existing measurement system.

shown in Fig. 2. Here a second directional coupler, sidearm power meter, and tuning transformer have been added. The assembly to be calibrated is enclosed by the dotted lines and $P_b$ has become $P_4$. These additional components permit one to explicitly account for nonideal impedance conditions in a way that leads to a determination of $K_A$ (rather than $K_c$) [6]. If $K_c$ is required, one may measure the reflection coefficient associated with the equivalent generator, $\Gamma_g$, as outlined in [4], and use the relation

$$K_c = K_A(1 - |\Gamma_g|^2). \tag{1}$$

For further details of these methods, and additional ones, the cited references should be consulted.

## IV. AUTOMATION CONSIDERATIONS

The measurement system of Fig. 1 is not generally considered suitable for automation because the deviations from idealized impedance conditions are only partially compensated for. Another system which has been briefly considered is shown in Fig. 2. Although the tuning adjustment does not lend itself to automation, this requirement may be eliminated in exchange for the phase difference between the detectors on arms 3 and 4, as in existing automated systems. It is possible to delete the tuner and devise a hybrid approach around this system using existing calibration schemes [2], [3], [5], provided one is willing to replace the bolometer mount by an alternative detector (as found in existing automated systems) during part of the calibration process. As a practical matter, one would like to eliminate the requirement for removal of the sidearm detector; this is achieved in the proposed methods.

## V. PROPOSED METHODS

### *A. Description*

One proposed measurement system is illustrated in Fig. 3. As before, the item to be calibrated is enclosed by dotted lines. The black box to the left represents the existing automated measurement schemes as described in [1]–[3]. Although these are generally geared to the measurement of power, little or no modification is required to permit a simultaneous measurement of impedance. Before measurements of either power or impedance can

[1] The reciprocal of this definition is also in use.
[2] Provided that the impedance of the Thevenin generator is equal to $Z_0$, an equivalent definition is possible in terms of the power "incident" upon a termination of arbitrary impedance.

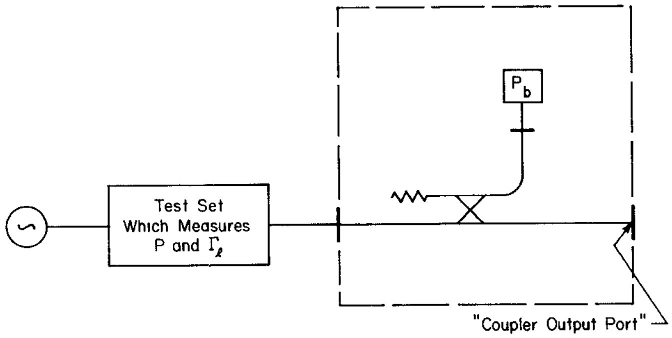


Fig. 3. Illustration of proposed measurement system.

be made by these existing systems, a "calibration" is first required. This calls for observing the system response to a suitably chosen set of power and impedance standards. In the proposed measurement procedure, *the directional coupler is treated as an extension of the output port of the automated measurement system*. By applying the calibration routine *at the output port of the coupler* one is able to measure both the power and impedance at this point. The remaining step is to compare the coupler sidearm power $P_b$ with either the incident or net power for a variety of values of load impedance. In actual practice, these may ordinarily be the terminations already used to calibrate the system. This permits one to compute $K_A$, $K_c$, and $\Gamma_g$. The mathematical details will be given in the following section.

## B. *Analytical Development*

For the purpose of this section it will be assumed that the "initial" calibration has been completed such that the power and impedance for any termination connected to the output port may be regarded as known quantities. The theoretical development begins with the well-known result

$$P_{gl} = \frac{P_g(1 - |\Gamma_g|^2)(1 - |\Gamma_l|^2)}{|1 - \Gamma_g\Gamma_l|^2} \tag{2}$$

where $P_{gl}$ is the net power delivered to a load of reflection coefficient $\Gamma_l$ by a generator having an available power $P_g$ and reflection coefficient $\Gamma_g$. By definition

$$K_cP_b = P_g(1 - |\Gamma_g|^2) \tag{3}$$

and it will prove convenient to let[3]

$$z = \frac{1}{P_{\text{inc}}} = \frac{1 - |\Gamma_l|^2}{P_{gl}} = \frac{|1 - \Gamma_g\Gamma_l|^2}{K_cP_b} \tag{4}$$

where $P_{\text{inc}}$ is equal to the power associated with the wave incident upon the termination $\Gamma_l$. Finally, let

$$u = \text{Re}(\Gamma_g)$$
$$v = \text{Im}(\Gamma_g)$$
$$x = \text{Re}(\Gamma_l)$$
$$y = \text{Im}(\Gamma_l). \tag{5}$$

With these substitutions, (4) may be written

$$1 - 2ux + 2vy + (x^2 + y^2)(u^2 + v^2) = K_cP_bz \tag{6}$$

or, alternatively, (if $u^2 + v^2 \neq 0$)

$$\left(x - \frac{u}{u^2 + v^2}\right)^2 + \left(y + \frac{v}{u^2 + v^2}\right)^2 = \frac{K_cP_bz}{u^2 + v^2}. \tag{7}$$

For a given value of $P_b$, the surface defined by (7) is a paraboloid of revolution, with axis parallel to the $z$ axis, and apex at the point $x = [u/(u^2 + v^2)]$, $y = -[v/(u^2 + v^2)]$, $z = 0$. It is easily demonstrated that if $|\Gamma_g| < 1$ the distance from the apex to the origin exceeds unity and becomes infinite as $|\Gamma_g| \to 0$. On the other hand, a passive termination ($|\Gamma_l| \leq 1$) requires $x^2 + y^2 \leq 1$. Thus the region of primary practical interest is that part of the surface defined by (7) which is bounded by the cylinder $x^2 + y^2 = 1$, as illustrated in Fig. 4.

## C. *Experimental Procedure*

Experimentally, the procedure is to observe the values of $P_b$, $x$, $y$, and $z$ for several terminations and fit an equation of the form of (6) to these measurement results. Because (6) contains three unknowns ($K_c$, $u$, $v$), a minimum of three equations, and thus three terminations, is required. The solution of the resulting system is not difficult (at least conceptually!). Since (6) is linear in $K_c$, this term may be easily eliminated between the three equations, yielding a pair of equations which is quadratic in $u$ and $v$. Ordinarily, a pair of quadratic equations has four sets of roots. However, because the coefficients of the $u^2$ and $v^2$ terms are equal, and the $uv$ term is absent in the resulting equations, it is immediately recognized that the loci are circles and that only two sets of roots are possible. These may be found by solving a quadratic equation. Although it is thus possible to write the general solution in closed form, this involves a substantial amount of algebra and has not been done. It is a fairly simple matter, however, to program a computer to follow the indicated steps. If the three terminations are properly chosen, it is possible to eliminate one set of roots on the basis of the tests $K_c > 0$ or $|\Gamma_g| < 1$. For example, if one uses three positions of a sliding short, the three points will lie on the curve represented by the intersection of the

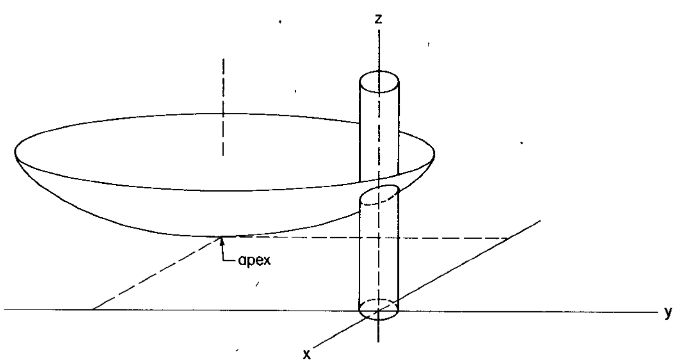


Fig. 4. Plot of (7).

[3] Note that when $|\Gamma_l| \to 1$, $P_{gl} \to 0$, but $z$ continues to be well defined.

parabolid and cylinder in Fig. 4, and the second solution will be a paraboloid whose apex is contained within the cylinder, which inplies $|\Gamma_g| > 1$.

In practice, it is unnecessary, and may be undesirable, to limit the terminations to a moving short. A detailed discussion of the geometry of this problem goes well beyond the scope of this paper, but the following conclusions have been developed with the aid of a time-share computer. In order to obtain a well-conditioned system of equations, and large separation between the two sets of roots, the triangle whose vertices coincide with the measurement points should ordinarily have the maximum possible area. This may be closely approximated by a sliding short where the plunger positions have been chosen to yield nominal phase-angle differences of 120°. An objection to this procedure is that it avoids the center of the permitted surface (corresponding to small values of $|\Gamma_l|$). If there are residual nonlinearities in the measurement system, it may be more desirable to choose a nominal impedance match for one of the terminations. In this case, the remaining short positions should have a 90° separation. Anything close to a 180° separation, or any other condition where the plane through the three points tends to approach a vertical position, must be avoided. As previously noted, it is frequently possible to use the observed response of the system during the initial calibration for the determination of $K_c$, $u$, and $v$ as well. However, one should examine the terminations to assure the foregoing conditions are satisfied. For example, if the only impedance standards used are the open circuit, short circuit, and matched termination, the system will be ill conditioned for $K_c$, $u$, and $v$.

Thus far, it has been assumed that only three terminations are to be employed. Although this represents a minimum for both the initial calibration, and for the determination of $K_c$, $u$, and $v$, there is a trend towards the use of more terminations in the initial calibration [5], [7]. Moreover, the standard power meter also represents a termination of arbitrary impedance, but which may be determined as a result of the "initial calibration." In general, it is desirable to use this entire collection of results in the $K_c$, $u$, $v$ determination. A convenient procedure for doing this is to solve the system of equations by the multidimension generalization of the Newton method which is described in texts on numerical methods. In this approach, (6) is replaced by its Taylor series expansion, in which only the first derivatives are retained. Because the resulting system is linear, the redundant data may be easily included and a standard least squares solution obtained. In order to apply this technique, an "initial estimate" of the solution is required. To obtain this estimate, it is convenient to assume $|\Gamma_g| = 0$ and take for $K_c$ the arithmetic average of the values obtained under the $|\Gamma_g| = 0$ assumption. Although the possibility of encountering an ill-conditioned system is greatly reduced with redundant data, the criteria previously outlined should still be kept in mind.

### D. Discussion

Perhaps the biggest drawback to the described approach is the large number of terminations, and thus operator effort, required. Generally speaking, the calibration calls for a power standard, an impedance standard, and either two more impedance standards or two sliding terminations of large and small reflection-coefficient magnitudes. This procedure must be repeated for each coupler. If the number of items to be calibrated is large, one may wish to consider an alternative method.

A second possible drawback, in some cases, is the relatively poor accuracy with which $\Gamma_g$ is determined, especially if $|\Gamma_g|$ is small. The reason for this may be recognized from Fig. 4. As previously noted, $\Gamma_g$ is determined by the position of the apex. If $|\Gamma_g|$ is small, the distance between the apex and the region where the experimental observations are made becomes large and the extrapolation suffers. Fortunately, however, the accuracy with which $K_c$ is determined is not affected by these considerations. Moreover, in many cases, interest in $\Gamma_g$ diminishes as $|\Gamma_g|$ becomes small, so the decreasing accuracy with which it is measured is not necessarily a problem.

## VI. ALTERNATIVE METHODS

Although the foregoing provides the basis for a practical measurement procedure, there are a number of alternatives which also warrant consideration. One of these is shown in Fig. 5. Here the output port of the coupler is connected to the output port of the measurement system, and the other coupler port terminates in a variable load. Prior to measuring the coupler, the test set is calibrated for power and impedance measurements at its output port *without* the coupler connected. With suitable reinterpretation, the

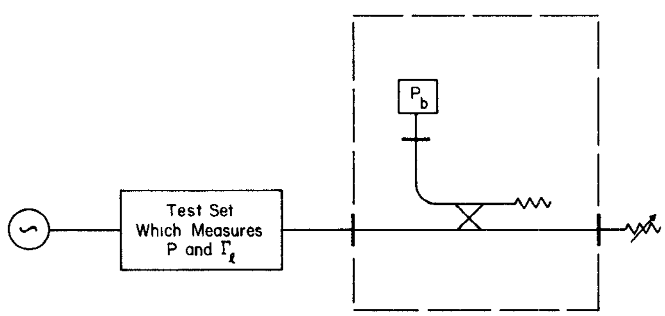


Fig. 5. An alternative measurement system.

prior theory may be applied. In particular, "$P_{\mathrm{inc}}$" for the coupler is now the *reflected* power observed by the measurement system. Moreover, "$\Gamma_l$" for the coupler is the *reciprocal* of that observed by the measurement system.

The advantage of this alternative procedure is that the "initial calibration" of the measurement system needs to be made only once. After this is done, the coupler and variable load are connected as indicated in Fig. 5. Perhaps the most convenient form of variable load is a sliding short (which need not be ideal). Three positions of the short provide enough data to permit a solution; four or more yield the advantages of redundancy and require little additional effort. If a large number of items are to be calibrated, it is immediately evident that this cailbration involves far less operator effort and calibration time.

Because of the reversed connection, the only values of "$\Gamma_l$" which are now possible are those of a magnitude greater than unity. In principle, with proper choice of variable load, this connection also makes it possible to explore the paraboloid in the neighborhood of its vertex, and thus measure $\Gamma_g$ with improved accuracy. Unfortunately, if this exploration is attempted, the signal level at the sidearm becomes quite small and may fall below the dynamic range of the sidearm detector. Even with the sliding short, the signal levels at the sidearm are less favorable than those for Fig. 3.

Perhaps the major drawback to this approach is the limitation of "$|\Gamma_l|$" to values greater than unity. By contrast, values of 0.1 and less are to be expected in the application of the coupler–power-meter assembly. As noted before, this application may aggravate the effect of residual nonlinearities in the system. However, until an experimental study can establish the validity of this objection, this alternative approach should be considered a strong candidate.

A further alternative is to exchange the positions of the generator and variable termination in Fig. 5. This alternative tends to combine some of the better features of the two prior methods. In this case, the "initial calibration" of the measurement system requires reexamination since it is now operating as a termination rather than signal source. Perhaps the largest drawback to this proposal is that it would require a modification of the existing automated measurement system. This modification does not appear to be warranted until the limitations of the prior methods have been experimentally evaluated.

## VII. POWER-EQUATION FORMULATION

Another approach to the problem, which provides a different set of advantages and limitations, is via the power-equation formulation [6].

As previously noted, the definition of $K_c$ involves a nonreflecting termination. This, in turn, calls for an impedance measurement capability which is reflected in the "initial-calibration" procedure. Following this, in addition to $K_c$, the described procedure also yields $\Gamma_g$ irrespective of whether or not it is explicitly required.

By contrast, $K_A$ is based upon the available power concept. From a practical point of view, the choice between $K_c$ and $K_A$ depends primarily upon the use envisioned for the coupler–power-meter unit. Past usage has been strongly oriented towards $K_c$, and provided one is willing to accept only a partial correction for mismatch, as explained earlier, this is the preferred parameter. Today, the trend is towards making a complete correction for mismatch effects; this correction is most conveniently done by the power-equation formulation [6], which also yields an insensitivity to certain connector problems [8]. Here, $K_A$ is the appropriate parameter and $\Gamma_g$ is irrelevant.

In the preceding discussion, the existence of both power and impedance standards have been assumed which, together with an appropriate procedure (of which there are many), permits a calibration of the test set in the automated measurement system. By contrast, on the basis of prior results from the power-equation formulation, one would expect to be able to measure $K_A$ using nothing more than a meter which measures net power, and a collection of offset or sliding shorts of unknown lengths. This measurement is indeed possible, as will now be shown.

A complete analytical development of the power-equation approach would be quite lengthy, and moreover would repeat much of the previously published material [9]. For this reason, the pertinent prior results will be briefly stated. For a more complete development, [9] should be consulted. In Fig. 6, that portion to the left of

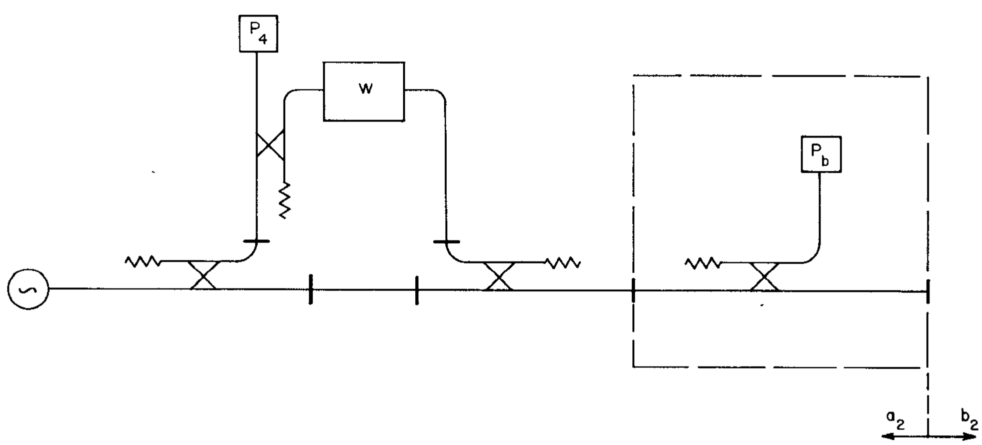


Fig. 6. A detailed measurement system.

the dotted line is a possible test-set configuration for measurement of both power and impedance. To the right is the coupler whose calibration is required. The response of the measurement system $w$, $P_4$, and that of the coupler under calibration, $P_b$, are related to the incident- and emergent-wave amplitudes by the equations

$$\Gamma_l = a_2/b_2 \tag{8}$$

$$w = \frac{a\Gamma_l + b}{c\Gamma_l + d} \tag{9}$$

$$P_4 = | ca_2 + db_2 |^2 \tag{10}$$

$$P_c = | Ca_2 + Db_2 |^2. \tag{11}$$

Here, $a$, $b$, $c$, and $d$ are determined by the parameters of both the measuring system and the coupler to be calibrated, while $C$ and $D$ are determined wholly by the latter. If the output port is terminated by an ideal moving short, the locus of $w$ in the complex plane is a circle of radius $r$, with center at $r_c$, where

$$r = \frac{| ad - bc |}{| d |^2 - | c |^2} \tag{12}$$

and

$$r_c = \frac{bd^* - ac^*}{| d |^2 - | c |^2}. \tag{13}$$

Let

$$w' = \frac{w - r_c}{r}. \tag{14}$$

Substitution of (9), (12), and (13) into (14) yields

$$w' = \frac{\Gamma_l - \Gamma_{gm}^*}{1 - \Gamma_l \Gamma_{gm}} e^{j\psi} \tag{15}$$

where

$$\Gamma_{gm} = -\frac{c}{d} \tag{16}$$

and

$$\psi = \arg [(ad - bc)/d^2]. \tag{17}$$

In line with the discussion found in [9], $\Gamma_{gm}$ is the reflection coefficient of the equivalent generator which obtains at the output port if $P_4$ is assumed to be constant.

Next, the ratio of (11) to (10) after being combined with (8) yields

$$\frac{P_b}{P_4} = \frac{| C\Gamma_l + D |^2}{| c\Gamma_l + d |^2} \tag{18}$$

and elimination of $\Gamma_l$ between (18) and (15) leads to

$$P_4(| D |^2 - | C |^2) | 1 - w'\Gamma_p |^2 = P_b(1 - | \Gamma_p |^2)(| d |^2 - | c |^2) \tag{19}$$

where

$$\Gamma_p = \frac{\Gamma_g - \Gamma_{gm}}{1 - \Gamma_g \Gamma_{gm}^*} e^{-j\psi} \tag{20}$$

and

$$\Gamma_g = - C/D. \tag{21}$$

Finally, it has been shown that [9]

$$K_A = (| D |^2 - | C |^2)^{-1} \tag{22}$$

and if $K_{Ag}$ represents the value of "$K_A$" for the "generator" associated with $P_4$ and $\Gamma_{gm}$

$$K_{Ag} = (| d |^2 - | c |^2)^{-1}. \tag{23}$$

Substitution of (22) and (23) in (19) gives

$$| 1 - w'\Gamma_p |^2 = P_b K_A (1 - | \Gamma_p |^2)/P_4 K_{Ag}. \tag{24}$$

By means of the methods described in [9], one can measure $r$, $r_c$, and $K_{Ag}$ with nothing more than a standard power meter and sliding short. This measurement constitutes the "initial calibration." In (24) one can observe $P_4$ and $P_b$, while $w'$ is obtained from the observed value of $w$ and the already determined $r$ and $r_c$. Thus the only remaining unknowns are $\Gamma_p$ and $K_A$. If (24) is further expanded, the resulting equation is of the same form as (6), and the earlier discussion is applicable.

Comparison with the earlier method indicates that the requirement for the impedance standard and the sliding load has been eliminated. Although this procedure calls for an "ideal" sliding short, the error expectancy from nonideal behavior is frequently small. In addition to $K_A$, the parameter $\Gamma_p$ is determined. From (20), however, it may be recognized that this parameter depends *both* upon the measuring system and the item under test. Further separation is not possible without a more elaborate "initial calibration."

Even where $K_c$ and $\Gamma_g$ are desired, the foregoing formulation provides a simplified computational procedure. In (24) $P_b$ and $P_4$ are observed parameters, while $w'$ is obtained from the observed $w$ with the help of (14). Once $K_A$ and $\Gamma_p$ are determined, $\Gamma_g$ may be obtained with the help of (20), (16), and (17), and $K_c$ may then be obtained from (1).

Alternatively, with the coupler reversed (Fig. 5), one has

$$| w' - \Gamma_q |^2 = P_b K_A (1 - | \Gamma_q |^2)/P_4 K_{Ag} \tag{25}$$

where

$$\Gamma_q = \frac{\Gamma_g - \Gamma_{gm}^*}{1 - \Gamma_g \Gamma_{gm}} e^{j\psi}. \tag{26}$$

The derivation of this result closely parallels that of (24) and will not be given, except to note that, in line with the discussion in conjunction with Fig. 5, $K_{Ag}$, $\Gamma_{gm}$, $\psi$, $r$, and $r_c$ now pertain only to the test set rather than the test set and directional coupler combination.

## VIII. SUMMARY

This paper has developed the theoretical basis for several different measurement schemes, each with a unique set of advantages and limitations. In order to provide a more definitive basis for choosing among them, an experimental evaluation appears necessary, but this has not been completed at this time.

As compared with the existing techniques for calibration of bolometer mounts, the methods proposed herein may appear unduly complex, at least in terms of operator effort. On the other hand, it must be recognized that the coupler–mount assembly is a two-port device, and to the author, at least, this increase in complexity appears unavoidable.

## ACKNOWLEDGMENT

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# Three-Dimensional Transmission-Line Matrix Computer Analysis of Microstrip Resonators

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***Abstract*—A wide range of microwave resonators are analyzed using the same three-dimensional transmission-line-matrix (TLM) computer program. The paper demonstrates the ease of application, versatility, and accuracy of the TLM method. The results presented include the dispersion characteristics of microstrip on dielectric and magnetic substrates and an example of a microstrip discontinuity. The surface-mode phenomenon of microstrip is also investigated.**

## I. INTRODUCTION

THE SOLUTION of large microwave integrated circuit (MIC) subassemblies presents a major problem to any numerical method. However, it would seem that the first step should involve a numerical routine of a very general nature for simple discontinuities in three-dimensional structures. There are many articles giving design data for single microstrip ([1]–[5], for example), pairs of coupled strips ([6]–[9], for example), and coplanar waveguides ([10] and [11]). Discontinuities which can occur in simple configurations such as abruptly ended strip conductor [12]–[14] and strip-width variation [12] have also been reported. Some of these publications use methods based on static approximations and all of them tend to use fairly specialized techniques and programs. Thus the design engineer does not have a universal and general program for solving a wide range of problems. The transmission-line matrix (TLM) [15], [16] method of numerical analysis in the form of a very general and short program fulfills this requirement.

The purpose of this paper is twofold: firstly, to review the general state of the art of the TLM method as far as the modeling of three-dimensional cavities is concerned; Secondly, to demonstrate the accuracy and the versatility

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